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Evolutionary Computing Approaches for Vehicle Routing and Scheduling Problems

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Extended Abstract

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Abstract

The thesis is concerned with the investigation of two vehicle routing and scheduling problems, namely the multiple traveling salesman problem (mTSP) and the vehicle routing problem with time windows (VRPTW), which is one of the most important and popular extensions of the vehicle routing problem (VRP). The multiple traveling salesman problem, a straightforward extension of the well-known traveling salesman problem (TSP), can be seen as a specific case of the vehicle routing problem, if removing the customer demands and vehicle capacity, involved in the formulation of the classical vehicle routing problem. Therefore, mTSP lies at the heart of many optimization problems found in transportation and logistics fields. Given the increasing need for mobility and its implications in logistics and supply chain management fields, that heavily rely on road network distribution, VRP, with mTSP at its core, attracted a great amount of research interest over the years, being intensively studied.

Since these two combinatorial optimization problems are NP-hard, being difficult to solve, and exact methods often require large computation times, which are unacceptable in real-life scenarios, metaheuristic approaches are most commonly employed for solving them. To this end, the thesis proposes a collection of ant colony optimization (ACO) based algorithms for solving two variants of the single-objective mTSP, one that aims to achieve balanced tours, and another one that aims to minimize the cost of the longest tour, achieving in this way an evenly distribution of the workload among salesmen. Moreover, the bi-objective mTSP, which seeks to simultaneously minimize the total traveled distance and the unbalancing degree of the tours, is also tackled in the thesis by relying on different ACO based multi-objective algorithms. Another study related with mTSP, examines the correlation between

the mechanism employed for selecting the salesman during the tour construction phase and the quality of the obtained solution, topic which was not investigated before in the literature.

The last part of the thesis is focused on the VRPTW problem, for which a collection of ACO algorithms is proposed. First, the single-objective VRPTW problem is addressed, which considers a hierarchical objective function, when we seek to minimize with priority the number of vehicles, and secondly we seek to minimize the total traveled distance. Next, the bi-objective VRPTW problem is investigated by means of a multi-objective ACO algorithm employing a decomposition scheme, when the two previous objectives are equally treated. Given its practical relevance, close to real-world contexts, the dynamic VRPTW is also studied in the thesis. This problem assumes the appearance during the working day of online requests from the customers, previously unknown in the system, that must be incorporated into an evolving schedule, whilst the vehicles are executing their routes.

To conclude, the thesis devises and examines a number of 23 ACO based algorithms, meant to tackle two important routing problems with wide practical applications, mTSP and VRPTW. The performed thorough experimental analysis of the proposed methods shows that they are able to obtain satisfactory results, providing in a limited amount of time solutions of good quality, which in some cases are competitive with other solutions reported in the literature. The ACO based algorithm designed for solving the bi-objective mTSP, outperforms other three multi-objective ACO algorithms proposed in the literature. Moreover, the algorithm developed for tackling DVRPTW, a problem which is closely related to real-world scenarios, proves to be superior to another ACO based algorithm from the literature, and it manages to found new best solutions for instances with a higher dynamicity, in which few customers' requests are known from the beginning, case more likely to occur in real-life situations. A benchmark of mTSP instances with reported (sub)optimal solutions computed with exact methods by the optimization software CPLEX is introduced in the thesis and made available online to aid researchers when comparatively evaluating their proposed heuristics. For evaluating the performance of the algorithms proposed for solving the VRPTW problem variants, the Solomon's 100 customers VRPTW test instances were employed, which is a well-known benchmark, frequently used in the literature, also available online. Besides this, in order to allow the interested reader to reproduce the reported results, the DVRPTW test instances and the source code of some algorithms are made freely available.

Contents

List of Publications

- [CONTR1] Raluca Necula, Mihaela Breaban, Madalina Raschip. *Tackling Dynamic Vehicle Routing Problem with Time Windows by means of ant colony system.* In 2017 IEEE Congress on Evolutionary Computation (CEC), pages 2480-2487, IEEE Press, 2017, doi:10.1109/CEC.2017.7969606
- [CONTR2] Raluca Necula, Mihaela Breaban, Madalina Raschip. *Tackling the Bi-criteria Facet of Multiple Traveling Salesman Problem with Ant Colony Systems.* In Proceedings of the 2015 IEEE 27th International Conference on Tools with Artificial Intelligence, ICTAI 2015, pages 873-880, IEEE Press, 2015, doi:10.1109/ICTAI.2015.127
- [CONTR3] Raluca Necula, Mihaela Breaban, Madalina Raschip. *Performance evaluation of ant colony systems for the single-depot multiple traveling salesman problem.* In E. Onieva, I. Santos, E. Osaba, H. Quintian, & E. Corchado (Eds.), Proceedings of the 10th International Conference on Hybrid Artificial Intelligence Systems, HAIS 2015, pages 257-268, volume 9121 of Lecture Notes in Computer Science, 2015, doi:10.1007/978-3-319-19644-2_22
- [CONTR4] Raluca Necula, Madalina Raschip, Mihaela Breaban. *Balancing the Subtours for Multiple TSP Approached with ACS: Clustering-based Approaches vs. MinMax Formulation.* In Proceedings of EVOLVE - A Bridge between Probability, Set Oriented Numerics, and Evolutionary Computation VI, EVOLVE 2015, pages 210-223, volume 674 of Advances in Intelligent Systems and Computing, Springer, 2018, doi:10.1007/978-3-319-69710-9_15

Thesis Overview

2.1 Motivation

Transportation plays an important role in our modern lives, as it covers a wide variety of situations ranging from the distribution of manufactured goods and provision of services in different parts around the globe to supporting the daily movement of people in various activities. Also, road transportation accounts as the prevailing manner of delivering and picking up goods worldwide. Two well-known combinatorial optimization problems associated with the transportation task are the traveling salesman problem (TSP) and the vehicle routing problem (VRP), which have roots in many real-life applications that can be naturally modelled as various TSP and VRP related problems. In connection with these two routing problems, the thesis is concerned with mTSP, a generalization of TSP, which employs several salesmen for visiting the given set of cities, and with VRPTW, an extension of the classical VRP, which considers in addition a time window constraint assigned to each customer. Most of the problems regarding transportation, that have a negative impact on our lives, are associated with an inefficient use of the available road infrastructure. Examples of such negative effects include congestion, increased pollution and more traffic accidents. Therefore, addressing these road transportation problems by investigating problems such as VRPTW and mTSP, a special case of VRP, is of uttermost importance since it can alleviate these issues and improve the quality of our lives. Besides this, the globalization of the world economy leads to a steadily increasing volume of goods that need to be transported between different locations in a timely manner. In order to remain competitive on the market, distribution companies need to streamline their operations and to use computer-aided systems that integrate optimization methods for supporting their decision making process. Thus, efforts need to be invested towards determining efficient route plans that diminish the incurred overall routing cost, while satisfying various constraints, aspect which implies solving different variants of the VRP problem. Moreover, finding optimal strategies in these routing problems helps to reduce the emissions that have a negative impact on the environment, and it mitigates delivery delays, which in turn increases the customers satisfaction.

The Vehicle Routing Problem has been widely studied for more than 50 years and thus, crystalized over the years as an active field of research. Also, VRP is one of the most important combinatorial optimization problem, having many practical applications in fields such as transportation and logistics, that cover a broad range of situations encountered in real-life, involving the distribution and collection of goods or people from one place to another. Besides this, the study of VRP has a practical importance, since it can yield worthwhile contributions in logistics systems, by reducing the associated costs, which leads VRP of having a considerable economical impact. As mentioned in Toth & Vigo (2001), by applying computerized methods to solve routing problems that arise in transportation, significant savings ranging from 5% to 20% regarding the total costs, can be achieved. This is more prominent in some market sectors, where transportation represents a high percentage from the end price of products.

Although is the core of VRP, and at the same time is a straightforward generalization of TSP, a well-known combinatorial optimization problem intensively studied over the years, mTSP received less research interest and applications of ACO algorithms for this problem were not widely investigated. mTSP can be regarded as an intermediate problem, which lays between TSP and VRP in terms of its involved complexity: mTSP is more difficult to solve than TSP, but at the same time, it is a more simpler problem compared to VRP, which incorporates additional constraints and aspects in its formulation. Moreover, mTSP lies at the heart of many problems arising in transportation and logistics fields and thus, it is worth studying this problem in order to devise efficient algorithms able to provide good quality solutions in a reasonable amount of time. The other problem examined in the thesis, the VRPTW, has a considerable practical relevance, being more closely related to real-world scenarios than the classical VRP, by integrating in its formulation the constraint of time windows, such that customers are available for delivery of services or goods only within a given time slot.

Since exact methods cannot be feasibly applied in practical situations to tackle difficult problems such as mTSP and VRPTW in a timely manner, we employed metaheuristic algorithms for their resolution. Over the years, swarm intelligence based approaches such as ACO algorithms have been successfully employed for solving many combinatorial optimization problems. Moreover, ACO algorithms can be naturally applied to problems which require finding the shortest paths on a graph, like the case of TSP, and also mTSP and VRPTW, which are generalizations of TSP. A set of algorithms based on the ACO metaheuristic are proposed and investigated in the thesis for tackling several variants of the mTSP and VRPTW problems, which are analyzed both as single-objective problems, but also from a multi-objective perspective. For most of the proposed approaches, the experimental results indicate that the ant colony based methods are efficient and are able to yield competitive results with the ones reported in the literature.

2.2 Contributions

The main contributions of the thesis can be summarized as follows:

- Given the fact that there are no freely available benchmarks for mTSP, as it is TSPLIB[1] for TSP, a benchmark of mTSP instances, available online as MTSPLIB[2], is introduced in the thesis, for which we report the obtained (sub)optimal solutions when solving these instances using exact methods by the optimization software CPLEX. The intent is that this benchmark to be reused by other researchers when evaluating the performance of their methods proposed for tackling mTSP.
- Five ACS based algorithms are developed and comparatively evaluated for tackling the single-objective bounded mTSP, which besides minimizing the total traveled distance, we seek to achieve balanced tours in terms of the number of cities visited on a salesman's tour, such that to achieve fairness among the workload assigned to each salesman (results published in paper [CONTR3]).
- In order to achieve balanced tours, we integrated clustering techniques such as K-means and Fuzzy C-means within several ACS based algorithms designed for the MinMax mTSP variant, aiming to partition in the best possible way the given cities into disjoint sets to be assigned to the salesmen. This mTSP variant although having practical relevance, was less studied in the literature, and to the best of our knowledge, few papers employed clustering algorithms when dealing with the MinMax mTSP variant (results published in paper [CONTR4]).

[1] http://comopt.ifi.uni-heidelberg.de/software/TSPLIB95/
[2] http://profs.info.uaic.ro/~mtsplib

- An ACO based algorithm following the MinMax approach is designed for tackling the bi-objective mTSP, which outperforms other multi-objective ACO algorithms proposed in the literature. This study aims to fill a gap in the literature, since few studies were identified that address mTSP from a multi-objective perspective. Moreover, the performed literature review revealed that there are no existing ACO approaches to tackle mTSP from an explicit bi-objective perspective (results published in paper [CONTR2]).

- A literature review on the existing schemes employed in the ACO based algorithms during the process of building the tours, when solving the mTSP problem.

- A set of ACO algorithms that employ different selection mechanisms of the salesman when building a solution for mTSP, and a new jointly selection scheme is proposed, which simultaneously chooses in a single step the salesman and the city to be visited next in its tour. The results are analyzed both from a single-objective and multi-objective perspective. To the best of our knowledge, there are no previous studies to treat mTSP at the same time from both aspects, and to analyze the correlation between the salesman selection scheme used in the solution construction process and the quality of the obtained solutions.

- A multi-objective ACO algorithm which relies on a decomposition scheme for tackling the bi-objective static VRPTW problem. Besides the fact that compared to the single-objective VRPTW, the multi-objective variant of VRPTW received less attention, to the best of our knowledge, there are no ACO approaches based on decomposition available in the literature for solving VRPTW from a multi-objective perspective.

- An ACO based algorithm which resorts to a joint solution construction mechanism for dealing with the single-objective dynamic VRPTW problem. Also, the proposed method integrates a local search procedure and an insertion heuristic, meant to further reduce the number of vehicles required to service the customers. The performed literature review revealed that few approaches that employ ACO algorithms were proposed for solving this VRPTW variant of significant practical relevance, being closely related to real-world scenarios. The experimental analysis showed that our method is competitive and it outperforms another ACO approach from the literature, managing to obtain better results especially on DVRPTW instances with a higher dynamicity level (results published in paper [CONTR1]).

- Besides devising and examining the aforementioned ACO algorithms for tackling mTSP and VRPTW problems, a significant effort was devoted to the implementation of these approaches. These proposed methods are developed in Java, and are based on the Java version of the ACOTSP[3] software package, which contains several ACO algorithms employed for solving symmetric TSP instances. The ACOTSP is a public software that provides a common framework that allows to implement different ACO algorithms for problems related to TSP. This aspect makes the source code to be easily reused for implementing other ACO algorithms and for enhancing the proposed methods with other constraints that may arise in mTSP and VRPTW problem variants. The ACO algorithms available in ACOTSP were adapted in the thesis such that to deal with the mTSP problem, and later on these algorithms were extended such that to take into account the additional constraints involved in the formulation of VRPTW. For the implemented algorithms that are made open source, we indicated in the experimental study in the corresponding section where they are described, the link to the repository where the source code can be found.

2.3 Structure of the Thesis

Chapter 1 provides the motivation underlying the research conducted in the thesis, concerning the two problems that are investigated: the Multiple Traveling Salesman Problem (mTSP) and the Vehicle Routing Problem with Time Windows (VRPTW). Then, it follows the enumeration of the contributions of the thesis, and finally, the organization of the thesis is presented.

Chapter 2 provides the theoretical background by outlining the ACO metaheuristic, and describes two ACO algorithms, the Ant Colony System (ACS) and the Max-Min Ant System (MMAS), upon which are based the approaches presented in Chapter 3 and 4 for tackling the mTSP and VRPTW problems, respectively.

Chapter 3 is devoted to the mTSP problem, by presenting its variants, the proposed benchmark of mTSP instances and by performing a survey on the literature concerning this problem. Also, several ACO based approaches are devised and investigated for dealing with different variants of the mTSP problem, that is examined as a single-objective and multi-objective problem.

[3] Thomas Stützle. ACOTSP, Version 1.0. Available from `http://www.aco-metaheuristic.org/aco-code`, 2004.

Chapter 4 addresses the VRP problem, by presenting its variants and by reviewing the related work regarding the VRPTW problem, an extension of the classical VRP. Since mTSP can be viewed as a simplified version of the classical VRP, some of the ACO algorithms proposed in Chapter 3 are extended and adapted in Chapter 4 for solving the VRPTW problem. As in case of mTSP, the designed ACO algorithms are meant to tackle VRPTW as a static single-objective and multi-objective problem, but in addition we also investigate VRPTW as a dynamic single-objective problem.

Finally, concluding remarks and future research directions are presented in Chapter 5.

2.4 ACO Approaches for the Multiple Traveling Salesman Problem

Although not so intensively studied as TSP, the mTSP is a straightforward extension of TSP of practical importance, requiring more than one salesman to be used for covering the whole set of cities. After giving the description of mTSP, and indicating its variants, a benchmark of mTSP instances is introduced in the thesis. Then, several studies, employing ACO algorithms, are presented that tackle the mTSP problem from a single-objective and multi-objective perspective. Designed for shortest path problems and with proven efficiency for TSP, ant colony algorithms are a natural choice for mTSP as well.

In Section 3.4.2 we comparatively evaluated the performance of several ACO based approaches for the balanced version of mTSP, in which we impose lower and upper limits for the number of cities to be visited in a tour, such as to achieve a fair distribution of the workload among salesmen. The results are reported to optimal solutions obtained on the mTSP benchmark from Section 3.3 using the optimization software CPLEX. This is a first attempt to publicly make available a benchmark for mTSP, along with reported (sub)optimal solutions computed on these instances with CPLEX. At the same time, this study aims to fill a gap found in the literature, given the fact that there is no investigation to evaluate the relative performance of several ACO based algorithms for mTSP. In Section 3.4.3 we also aimed to balance the tours of salesmen, but in terms of the cost (distance) of each tour. For this, we employed the MinMax formulation of mTSP, that although having a greater practical importance, was less studied in the literature. In order to achieve a balanced workload, the proposed methods incorporate clustering techniques, which were not considered before in a combination with a swarm intelligence algorithm for solving the MinMax mTSP variant. As an extension of the previous two studies, which consider mTSP as a single-objective problem, we tackled next the mTSP from a multi-objective perspective, given the fact that few previous articles addressed the bi-criteria facet of mTSP in an explicit manner. Thus, we investigated in Section 3.5.2 three multi-objective ACO algorithms and a single-objective one, and analyzed their performance according to the quality of their constructed approximations of the Pareto front. Section 3.6 examined the correlation between the mechanism employed to select the salesman during the tour construction phase, and the quality of obtained solutions, aspect which was not investigated so far in the literature. In this sense, we proposed three schemes, among which two are sequential ones, with the salesman being selected in a random and in a probabilistic manner, respectively, and the last one, being a jointly selection scheme, in which the salesman and the city to be visited in his tour, are simultaneously selected at the same step. Compared to existing approaches in the literature, we evaluated the obtained solutions in a twofold manner, such that the solutions are analyzed both from a single-objective perspective, given the MinMax objective, but also under a bi-criteria context, as performed in Section 3.5.2.

2.5 ACO Approaches for the Vehicle Routing Problem

Vehicle Routing Problem (VRP) is a widely studied problem in the field of Operations Research, due to its great applicability in supply chain management, which involves the physical delivery and/or collection of goods or services from customers. Also several works, conducted in the thesis regarding a VRP variant, namely the Vehicle Routing Problem with Time Windows (VRPTW), which is one of the most studied variants of VRP, are further described. When every customer has an associated time window, we are dealing with the VRPTW, an NP-hard problem extensively studied, which is one of the most popular vehicle routing problem type. In VRPTW several customers must be served within their specified time interval, and the schedule of the vehicle routes needs to be established. The methods proposed can be seen as extensions for the VRPTW problem of the ACO algorithms devised in Chapter 3 for the mTSP problem. This statement is justified by the fact that mTSP can be regarded as a simplified version of the (classical) VRP, which assumes only vehicles capacities as additional constraint. If the vehicles' capacities are sufficiently large in VRP such as to not restrict the vehicles' capacities, then VRP is the same problem as mTSP, and therefore mTSP can be considered as uncapacitated VRP.

Section 4.3 presents a preliminary work in devising ACO algorithms that deal with the single-objective static VRPTW problem. In this sense, three ACS based approaches are proposed and examined, one of them including an insertion heuristic combined with local search operators, phase intended to further reduce the number of vehicles necessary for serving all the customers. The evaluation of the investigated algorithms is performed considering an hierarchy between the two objectives: the minimization of the number of vehicles (or number of tours) takes precedence over the minimization of the total traveled distance, approach which is frequently employed by the heuristics proposed in the literature for solving VRPTW. The experimental analysis is conducted on several instances from the Solomon's classic VRPTW 100 customers benchmark, and shows promising results compared with another ACO algorithm and with the best-known solutions available in the literature. However, additional mechanisms need to be incorporated in order to improve the performance of the proposed algorithms, and to obtain competitive results with the ones reported in the literature.

As a continuation of this study, we consider in Section 4.4 the same static VRPTW problem, but treated under a multi-objective context, when both objectives, the number of vehicles and the total distance, have the same importance and no priority between them is assumed. Also, the study from this section aims to fill a gap existing in the literature, given the fact that compared with the single-objective VRPTW, fewer articles are available that address the multi-objective VRPTW, despite this version is able to better capture problems found in real-life. To this end, we devised a multi-objective ACS based algorithm which relies on a decomposition scheme, which is, to the best of our knowledge, the first ACO approach in the literature based on the decomposition concept that deals with the multi-objective VRPTW. The computational study shows that our algorithm is able to yield promising results, however in order to achieve competitive results with another decomposition based method from the literature, it is necessary to incorporate an additional mechanism aimed to enhance the diversity of solutions.

Since most real-world routing problems involve a dynamic aspect, in Section 4.5 we investigate the single-objective dynamic VRPTW by means of ACS based algorithms. We consider that the dynamic nature of the problem is given by the customers' requests, in which not all of them are known beforehand, but some of them are revealed dynamically during the working day, while the vehicles are executing their routes. The aim of this study is to design and analyse ant colony optimization based algorithms, that are able to achieve good quality solutions for DVRPTW instances with a higher level of dynamicity. To this end, we propose an ACS based algorithm, relying on a joint mechanism for constructing the solutions, in which the vehicle and the next customer to be added to its tour are simultaneously selected during the transition step. Our approach is hybridized with a local search procedure consisting of two operators, relocate and exchange, that are applied in an iterative manner, until no further improvement is possible. Moreover, we integrated in our method an insertion heuristic to better incorporate unvisited customers in the existing tours, thus reducing the number of vehicles needed to service all the customers. The experimental study reveal that our method is able to generate good quality solutions especially for DVRPTW instances with a higher dynamicity, outperforming another ACS based approach from the literature.

Conclusions and Future Work

The thesis focused on studying two well-known combinatorial optimization problems, mTSP, which is a generalization of TSP, and VRPTW, having mTSP at its core, by relying on metaheuristic methods. VRPTW is one of the most studied variants of VRP, fact proved by the vast amount of literature covering this problem. Given their considerable difficulty and inherent practical applications, which arise in transportation and logistics fields, these problems have attracted a lot of research interest over the years, crystallizing into important topics in the Operations Research area. Also these two routing problems are ubiquitous in many industries, since transportation is involved in every aspect of our modern lives. Besides this, these problems have a considerable economic impact, since they entail a significant part of the operational cost of many companies. It was shown that devising computerized methods for solving such kind of problems can bring significant savings in the costs incurred by distribution companies and in the final price of products. Thus, it is worthwhile studying these problems, and designing effective and efficient algorithms aimed to tackle them.

Due to their high involved computation time, most of the exact methods are prohibitive when being applied to large size mTSP and VRPTW instances that typically arise in real-world scenarios. In these cases, exact methods are not guaranteed to reach to an optimal solution, and it is more appropriate to apply metaheuristics that can provide approximate near optimal solutions in a limited amount of time, such that they can be successfully implemented for solving real-life problems. Therefore, several approaches based on the ACO metaheuristic, which fits into the swarm intelligence framework, were devised in the thesis for solving mTSP and VRPTW problems. The ACO metaheuristic is a population-based method, which draws its inspiration from the foraging behaviour observed in real ant colonies, that succeed in finding short paths between their nest and food sources.

First, we investigated the mTSP problem from a single-objective and multi-objective perspective. When treating mTSP as a single-objective problem, we aimed to minimize the total traveled distance and to achieve balanced tours in terms of the number of cities to be traveled in each salesman tour. We also sought to achieve balanced tours, by minimizing the cost of the longest tour, when we included in our approaches clustering techniques for dividing the cities into equal groups. When interpreting mTSP in a multi-objective context, two objectives were simultaneously optimized: the total cost and the balancing degree of the tours, which was measured as the difference between the cost of the longest tour and the cost of the shortest tour. Then, we examined the impact of the salesman selection scheme, used during the solutions construction phase, on the quality of generated solutions, when the proposed methods were evaluated both from a single-objective and bi-objective perspective.

As in case of mTSP, when investigating VRPTW, we analyzed it first as a static single-objective and multi-objective problem, and then as a dynamic single-objective problem. To this end, we extended and adapted some of the algorithms designed in the thesis for tackling mTSP, given the fact that VRPTW is a generalization of mTSP. When addressing the static single-objective VRPTW, we considered a hierarchical objective function, commonly employed by the heuristics available in the literature for VRPTW, which assumes a priority among the two objectives, the number of used vehicles and the total traveled distance. This involves that minimizing the number of vehicles is the primary objective, whilst the secondary objective is to minimize the total traveled distance. Then, we studied the same static VRPTW, but interpreted from a multi-objective perspective, since most of the real-world routing applications possess an inherent multi-objective nature. In this case, we aimed to simultaneously optimize the two previous objectives, which were equally treated, having the same importance. Lastly, in an attempt to come closer to the real-world scenarios, we took into account the real-time aspect of VRPTW, when we analyzed it as a single-objective problem, considering the same hierarchical objective function. We assumed that the dynamic aspect of VRPTW is given by the customers' requests, when not all of them are known beforehand, but some of them are dynamically revealed as the working day progresses.

The experimental analysis showed that overall our proposed methods are capable of obtaining promising results, providing good values for the considered objectives, and yielding approximation sets that offer diverse trade-off solutions in proximity of the reference Pareto front. In case of the single-objective bounded mTSP and MinMax mTSP variants, one of the best solutions are achieved by the ACO based algorithms which impose bounds on the size of the tours and by the ACO based algorithm which follows the MinMax approach, respectively. Also, in case of larger mTSP instances, with high number of cities and many salesmen, the method which integrates the K-means clustering technique, might prove useful due to its ability to decompose the initial problem instance into a number of smaller subproblems. When considering the VRPTW problem, the approaches that coupled the ACO algorithms with local search and insertion heuristics procedures seem to provide better results. Additionally, in case of dynamic optimization problems, such as DVRPTW, the use of a joint mechanism for building the vehicles tours, which gives better chances to incorporate the available unvisited customers into the existing vehicles tours, helps to obtain high quality solutions, in a limited amount of time, such that it can be successfully applied in practical scenarios.

In conclusion, the thesis gave an insight into the applicability of natural computing techniques for tackling difficult routing and scheduling problems. More precisely, it showed how ACO based algorithms can be designed for solving two challenging routing problems, mTSP and VRPTW, of significant practical importance, which fits many real-world scenarios from transportation and logistics fields. The computational results indicated that unlike exact methods, ACO based approaches are able to provide good quality solutions in a reasonable amount of time, and might prove a viable option when addressing real-world optimization problems.

Possible avenues for future work include solving the DVRPTW problem from a multi-objective perspective and extending the research to tackle dynamic rich vehicle routing problems (Lahyani et al., 2015), that can incorporate more complex constraints and objectives, found in real-life VRPs. Among the additional attributes to be considered in the formulation of the problem, we may refer to: time-dependent travel times, heterogeneous fleet of vehicles, multiple depots, multiple routes per vehicle, pickup and delivery operations associated with each customer location, and open routes as in case of open VRP, which do not require vehicles to return to the depot, once they completed their tour. Also, another research direction is to study the car pooling problem and the dynamic ride-sharing problem (Agatz et al., 2012; Furuhata et al., 2013), which are related with the VRP and dial-a-ride problem.

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List of Abbreviations

ACO Ant Colony Optimization. iii

mTSP Multiple Traveling Salesman Problem. 6

TSP Traveling Salesman Problem. iii, 2

VRP Vehicle Routing Problem. iii, 2, 8

VRPTW Vehicle Routing Problem with Time Windows. 6, 8