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An Event Based Semantics of P Systems

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Abstract

Membrane systems have many similarities with classical concurrency models. In particular notions like *parallelism*, *causality* and *concurrency* seem to belong to membrane computing, though they are not yet regarded as central or cornerstone notions. Recently the interest in comparing membrane systems and other models for concurrency has grown. In this paper we propose a translation of membrane system into zero safe nets and then we show how to associate an event automaton to the 1-unfolding of these nets. Thus we propose an event based view of computations of a membrane system.

1 Introduction

P systems, or membrane systems, introduced in [12, 13], have, among others, the aim of giving a formal modeling to the structure and the functioning of the cell.

Membrane systems are based upon the notion of membrane structure, which is a structure composed by several membranes, hierarchically embedded in a main one, called the skin membrane. A plane representation of a membrane structure can be given by means of a Venn diagram, without intersected sets and with a unique superset. The membranes delimit regions (compartments) and to each region we associate a (multi)set of objects, described by some symbols over an alphabet, and a set of evolution

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rules, which can modify the objects to obtain new objects and possibly send them outside the membrane or to an inner membrane. The various compartments have different tasks, and all together they contribute to accomplishing a more complex one. In the basic variant the evolution rules are applied by just checking whether the objects to transform are present, in the variant we consider in this paper also *promoters* and *inhibitors* are taken into account, i.e., *side* conditions stating when an evolution can succeed or is forbidden, as introduced in [3]. The evolution rules are usually applied in a maximally parallel manner: at each step, all the objects which can evolve should evolve. If we start from an initial configuration, with a certain number of objects in certain membranes, and we let the system evolve, we obtain a computational device. If a computation halts, that is no further evolution rule can be applied, the result of the computation is defined to be the number of objects in a specified membrane (or expelled through the skin membrane). If a computation never halts (i.e., one or more objects can be rewritten forever), then it provides no output.

Recently the interest in capturing notions like *concurrency* and *causality* in membrane systems has arisen (e.g., [6, 8]). In fact in membrane systems parallelism and concurrency are present, and in systems exploiting these notions, causality is a central concept. To understand what dependency means in this setting, we relate membrane systems with more classical concurrency models, and thus we can try to identify what *events* in membrane computing are.

In [10, 11] a comparison with a suitable class of nets, Place/Transition nets with explicit localities, is used to capture the compartment structure of membrane systems. Each locality identifies a distinct set of transitions which may only be executed synchronously, i.e., in a locally maximal concurrent manner; and a notion of process for these nets is developed, with the associated notion of dependency. This is further studied and cast in a more general framework in [9]. Busi in [6] studied the causal dependencies by introducing a notion of reaction which can be decorated with names, and in [8] Ciobanu and Lucanu studied which kind of event structure arises from membrane systems. Here the interesting feature is the attempt to identify differently what an *event* is in membrane computing, not as single occurrence of rules but as the computational entity changing the state.

Following the lines of thought commenced in these papers, we first introduce a Petri net view of membrane systems which is, differently from other approaches, based on zero safe nets and on the collective token philosophy. Zero safeness takes into account the compound nature of the evolution step, based on the application of several rules. As zero safeness allows to synchronize transitions in nets, it seems to be the correct notion to be used in this setting. In particular, zero safe nets allow to define in a cleaner way what the application of a set of rules in a membrane system is, using zero safe places to represent partial configurations, introduced by Busi in [7], which are useful in capturing dependencies among rules applications.

The collective token philosophy in Petri nets captures in a more precise way the multiset approach of membrane computing, as it allows to partly forget dependencies among steps leading to the same result. We illustrate this with two examples.

First consider the P system

$$(\{a, b, c\}, [_1\ [_2\ ]_2\ ]_1, c, aa, \{r_1 = bc \to (a, here)\}, \{r_2 = a \to (b, out)\})$$

with three objects (a, b and c), two membranes, two sets of rules, the one associated to the external membrane (r_1) transforming the multiset with one occurrence of b and one of c in the multiset with just one a, and the set of rules associated to the internal membrane contains one rule (r_2) transforming the multiset with one occurrence of a in the multiset with one occurrence of b which is sent to the outer membrane. The initial state is the multiset with just one occurrence of c in the external membrane and the multiset with two occurrences of a in the internal one. In the first step two instances of the rule r_2 are used, and the new state is the multiset with two occurrences of b and one of c in the external membrane and the empty multiset in the internal one. Now just one instance of the rule r_1 can be applied, and the collective token philosophy allows to say that r_1 depends on one of the instances of r_2, ignoring which is the one that actually produced the used instance of the object b. Hence the collective token philosophy reflects the anonymity of multiset rewriting.

Second consider the P system with promoters and inhibitors (i.e., side conditions to be tested to apply rules)

$$\begin{gathered}(\{a, b, c\}, [_1\ [_2\ ]_2\ ]_1, ab, \mathbf{0}, \\ \{r_1 = a \to_b^\emptyset (a, in_2), r_2 = a \to_\emptyset^c (a, in_2)\}, \{r_3 = a \to_\emptyset^\emptyset (c, here)\})\end{gathered}$$

with three objects (a, b and c), two membranes, and two sets of rules. Only one of the two rules of the first set can be applied at the initial state, and furthermore the two rules produce the same effects, but they test different side conditions (r_1 can be applied if b is present, whereas r_2 can be applied if

c is absent. The effects of the application of each of these are the production of an a in the membrane 2, and then the rule r_3 can be applied. Clearly the application of the rule r_3 (i.e., the event associated to it) depends either on r_1 or on r_2 and this kind of *or* causality can be captured by the collective token philosophy, whereas in the individual token philosophy we would have two *different* events associated to r_3, one depending on the one associated to r_1 and the other depending on the one associated to r_2. Again the collective token philosophy allows to say that r_3 depends on the production of certain objects in the external membrane, ignoring which is the one that actually produced the used instance of the object a.

Within the collective tokens philosophy the behaviour of a net is viewed as a 1-occurrence net ([17, 18] and [14]), where again, differently from the more classical notion of occurrence net, a transition may happen (partially) ignoring which other transition has produced tokens. The unfolding of a net into the corresponding 1-occurrence net is straightforward, and it is easy to associate to the state of these nets (which are suitable subsets of transitions) an event automaton. Thus in this paper we identify the events as the actual application of rules of the membrane system, differently from [8]. However, this is a fine grained view, which we consider to be a starting point rather then the final target of an investigation of what an event in membrane systems should be.

The paper is organized as follows: in the next section we fix some notation and then, in section 3 we review the notion of P system. In section 4 we present the relevant notions about zero safe nets, occurrence nets and 1-unfolding, and then, in section 5 we relate membrane systems and nets. In section 6 we recall the notion of event automata and we show how to associate an event automaton to the states of the 1-unfolding of a net corresponding to a membrane system, and we finally discuss how a more precise relation capturing dependencies can be introduced.

2 Background

We fix the notation that will be used in the paper. Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{N}^+ = \mathbb{N} \setminus \{0\}$. Given a set S, a *finite multiset* over S is a function $m : S \to \mathbb{N}$ such that the set $dom(m) = \{s \in S \,|\, m(s) \neq 0\}$ is finite. The *multiplicity* of an element s in m is given by $m(s)$. The set of all finite multisets over S, denoted by $\mathcal{M}_{fin}(S)$, is ranged over by m. A multiset m such that $dom(m) = \emptyset$ is called *empty* and it is denoted by

0. With $\mathbf{2}^S$ we denote the set of the subsets of S. The set of all finite sets over S is denoted by $\mathbf{2}^S_{\mathit{fin}}$, and $\mathbf{2}^S_1$ are those with at most one element. The cardinality of a multiset is defined as $|m| = \sum_{s\in S} m(s)$. We write $m \subseteq m'$ if $m(s) \leq m'(s)$ for all $s \in S$, and $m \subset m'$ if $m \subseteq m'$ and $m \neq m'$. The operator $\oplus$ denotes *multiset union*: $m \oplus m'(s) = m(s) + m'(s)$. The operator $\setminus$ denotes *multiset difference*: $m \setminus m'(s) =$ if $m(s) > m'(s)$ then $m(s) - m'(s)$ else 0. The *scalar product* of a number j with a multiset m is $(j \cdot m)(s) = j \cdot (m(s))$.

The language of *membrane structure*, denoted with MS, is a language over $\{[,]\}$ whose strings are defined as follows: (i) $[\,] \in MS$, (ii) if $\mu_1, \ldots, \mu_n \in MS$, with $n \geq 1$, then $[\mu_1 \ldots \mu_n] \in MS$, and nothing else is in MS. The same membrane structure can be represented by several equivalent strings (the equivalence being that $\mu_1\mu_2\mu_3\mu_4 \equiv \mu_1\mu_3\mu_2\mu_4$, for $\mu_1\mu_4 \in MS$ and $\mu_2, \mu_3 \in MS$) hence we assume that the canonical representation of it is given by a tree like structure, i.e., a membrane structure is a rooted tree. We call *membrane* each matching pair of parentheses appearing in the membrane structure.

Given a membrane structure μ, the number of (nested) membranes is defined as follows: $\mathit{mem}([\,]) = 1$ and $\mathit{mem}([\mu_1 \ldots \mu_n]) = \sum_{i=1}^n \mathit{mem}(\mu_i) + 1$. The depth of a membrane structure $\mu \neq [\,]$, i.e., the maximal number of nested membranes, is easily defined as $\mathit{depth}([\,]) = 1$ and $\mathit{depth}(\mu) = \mathit{max}\{\mathit{depth}(\mu_i) \mid \mu = [\mu_1 \ldots \mu_n],\ 1 \leq i \leq n\} + 1$. The depth of the membrane $[\,]$ is equal to 0. Given a membrane μ, to each nested membrane it is possible to associate a unique index (from 1 to $\mathit{mem}(\mu)$), hence we freely identify a membrane with an index (the convention being that μ has the number 1, and if i is the index of a membrane μ_i and j is the index of a nested membrane of μ_i, then $j > i$). Given a membrane i, $i \neq 1$, it has a father, which is the membrane j, $j \leq i$, such that $\mu_j = [\ldots \mu_i \ldots]$. The function $\mathit{father}(i)$ returns the index of the father membrane if $i > 1$ and it is undefined otherwise (thus the father of the outer membrane does not exist). A membrane j can have children, i.e., membranes $j_1, \ldots, j_k$ such that $\mathit{father}(j_1) = \ldots = \mathit{father}(j_k) = j$. Hence the function *children* returns a set of indexes $\mathit{children}(i) = \{j \mid \mathit{father}(j) = i\}$. A membrane structure can be seen not only as a rooted tree, but is often represented as Venn diagram in which any closed space (delimited by a membrane and by the membranes immediately inside) is called a *region* (or compartment).

3 P Systems

In this section we recall the definition of *membrane systems*. We will consider the class of membrane systems with *promoters* and *inhibitors*.

Definition 1 *A* membrane system with promoters and inhibitors *over* V*, a finite alphabet of (names of) objects or molecules, is a construct* $\Pi = (V, \mu, w_1^0, \ldots, w_n^0, R_1, \ldots, R_n)$ *where:*

- μ *is a* membrane structure *with* n *membranes indexed* $1, \ldots, n$*,*
- *each* w_i^0 *is a multiset over* V *associated with membrane* i*, and*
- *each* R_i *is a finite set of* reaction (or evolution) rules r *associated with the membrane* i*, of the form* $u \rightarrow_{inh}^{p} v$*, where* u, p, inh *are finite multisets over* V*, and* v *is a finite multiset over* $V \times (\{here, out\} \cup \{in_j \mid father(j) = i\})$*, and each rule is such that* $u \neq \mathbf{0}$*.*

Given a rule r, u is the left hand side of r, p and inh are respectively the *promoters/inhibitors* of r, and v is the right hand side of r. To ease the notation, given a rule $r = u \rightarrow_{inh}^{p} v$, with $\pi(v)|_\alpha$ we denote the multiset on V obtained from v by considering all the elements with the second component equal to α. In the following we will often omit $(V, \mu, w_1^0, \ldots, w_n^0, R_1, \ldots, R_n)$ when it is possible and no confusion arises, and indicate a membrane system simply with Π. We will also omit to say explicitly that the membrane systems we consider are with promoters and inhibitors, as this will always be the case.

A membrane system Π evolves from configuration to configuration as a consequence of the application of (multisets of) evolution rules in each region. We start formalizing the notion of configuration of a membrane system. Following Busi [6] and [7], we introduce the notion of partial configuration, which captures the following idea: the state of each membrane is divided in two parts, what can be used (consumed or tested), and what is produced *during* the evolution, pointing out in a clearer way the effects of each rule application.

Definition 2 *Let* Π *be a membrane system, then a* configuration *is a tuple* $C = (w_1, \ldots, w_n)$ *where each* w_i *is a multiset over* V*.* $C_0 = (w_1^0, \ldots, w_n^0)$ *is the* initial *configuration of* Π*. The set of configurations of a membrane system is denoted with* $Conf_\Pi$*.*

A partial configuration *is a tuple* $C = ((w_1, \overline{w}_1), \ldots, (w_n, \overline{w}_n))$ *where each* $w_i, \overline{w}_i$ *is a multiset over* V. $C_0 = ((w_1^0, \mathbf{0}), \ldots, (w_n^0, \mathbf{0}))$ *is the* initial *partial configuration of* Π. *The set of partial configurations of a membrane system is denoted with* $PConf_\Pi$.

To each configuration $(w_1, \ldots, w_n)$ a partial configuration corresponds, namely $((w_1, \mathbf{0}) \ldots, (w_n, \mathbf{0}))$. A configuration is obtained from a partial one by simply adding, for each pair $(w_i, \overline{w}_i)$, to the left hand side w_i, the right hand side $\overline{w}_i$, and then by setting each right hand side to $\mathbf{0}$) and then forgetting the right hand sides[3].

Definition 3 *Let* Π *be a membrane system, and* $((w_1, \overline{w}_1), \ldots, (w_n, \overline{w}_n))$ *be a partial configuration. Then* $heated(((w_1, \overline{w}_1), \ldots, (w_n, \overline{w}_n))) = (w_1 \oplus \overline{w}_1, \ldots, w_n \oplus \overline{w}_n)$.

We formalize the evolution of a membrane system in a slightly different way with respect to the classical approach. We proceed in two stages: first we define a partial reaction relation (which is basically a one step relation) describing the effects of the actual application of a rule and then, using this one, we define the evolution of the whole system.

Definition 4 *Let* Π *be a membrane system, and let* $r = u \to_{inh}^{p} v \in R_i$ *be a rule. The* partial reaction relation $\overset{r}{\mapsto} \subseteq PConf_\Pi \times PConf_\Pi$ *is defined as follows:* $\gamma = ((w_1, \overline{w}_1), \ldots, (w_n, \overline{w}_n)) \mapsto ((w'_1, \overline{w}'_1), \ldots, (w'_n, \overline{w}'_n)) = \gamma'$ *whenever*

- $w'_i = w_i \setminus u$ *and* $\overline{w}'_i = \overline{w}'_i \oplus \pi(v)|_{here}$,
- $\forall j \neq i \; w'_j = w_j$,
- *for* $i \neq 1$, $\overline{w}'_{father(i)} = \overline{w}_{father(i)} \oplus \pi(v)|_{out}$, $\forall j \in children(i)$, $\overline{w}'_j = \overline{w}'_j \oplus \pi(v)|_{in_j}$, *and*
- $\forall j \; j \neq i$ *and* $j \notin children(i)$ *and* $j \neq father(i)$ *it holds that* $\overline{w}'_j = \overline{w}_j$.

If the same evolution rule r *is used more then one time (h times) we write* $\gamma \overset{r}{\mapsto}_h \gamma'$ *to indicate that there are partial configurations* $\gamma_1, \ldots, \gamma_{h-1}$ *such that* $\gamma \overset{r}{\mapsto} \gamma_1 \overset{r}{\mapsto} \ldots \overset{r}{\mapsto} \gamma_{h-1} \overset{r}{\mapsto} \gamma'$.

[3] This operation is called *heated*$(\cdot)$ in [6].

Finally we formalize the more classical notions of evolution of a membrane system. Let Π be a membrane system, then a *vector multi-rule* $\vec{R}$ is the n-tuple $(\widehat{R}_1, \ldots, \widehat{R}_n)$ where $\widehat{R}_i$ is a multiset over R_i. With $r_i^j = u_{ij} \rightarrow_{inh_{ij}}^{p_{ij}} v_{ij}$ we indicate the j-th rule in R_i. We stress that the applicability test of a vector multi-rule is done before applying the rules.

Definition 5 *Let* Π *be a membrane system, and let* $\gamma = ((w_1, \overline{w}_1), \ldots, (w_n, \overline{w}_n))$ *be a partial configuration. Let* $\vec{R}$ *be a vector multi-rule, then* $\vec{R}$ *is* enabled *at* γ *iff for all* i, $\sum_j \widehat{R}_i(r_i^j)(u_{ij}) \subseteq w_i$, *for all* i, j, $p_{i,j} \subseteq w_i$, *and* $w_i(a) < inh_{i,j}(a)$ *for all* $a \in dom(inh_{i,j})$.

Let $\vec{R}$ be a vector multi-rule, with $lin(\vec{R})$ we denote a linearization of it, i.e., a sequence of rules r_i^j where for each rule r_i^j there are exactly $\widehat{R}_i(r_i^j)$ occurrences. We indicate such sequence, with abuse of notation, with $\widehat{R}_1(r_1^1), \ldots, \widehat{R}_1(r_1^{l_1}), \ldots, \widehat{R}_n(r_n^1), \ldots, \widehat{R}_n(r_n^{l_n})$.

Definition 6 *Let* Π *be a membrane system. The* reaction relation $\Rightarrow \subseteq Conf_\Pi \times Conf_\Pi$ *is defined as follows:* $(w_1, \ldots, w_n) \Rightarrow (w'_1, \ldots, w'_n)$ *iff there exists a* vector multi-rule $\vec{R}$, *a linearization* $lin(\vec{R}) = \widehat{R}_1(r_1^1), \ldots, \widehat{R}_n(r_n^{l_n})$ *and partial configurations* $\gamma_0, \ldots \gamma_m$ *such that:*

- $\vec{R}$ *is enabled at* $\gamma_0 = ((w_1, \mathbf{0}), \ldots, (w_n, \mathbf{0}))$,
- $m = \sum_{i=1}^n |\widehat{R}_i|$ *and* $(w'_1, \ldots, w'_n) = heated(\gamma_m))$, *and*
- $\gamma_0 \overset{r_1^1}{\mapsto}_{\widehat{R}_1(r_1^1)} \gamma_{\widehat{R}_1(r_1^1)} \cdots \gamma_{m - \widehat{R}_n(r_n^{l_n})} \overset{r_n^{l_n}}{\mapsto}_{\widehat{R}_n(r_n^{l_n})} \gamma_m$.

We stress that the order of evolution rules application is inessential, as their applicability is tested at the beginning, hence there are many linearizations of the same vector multi-rule, and all of them will lead to the same result.

We are now ready to define the notion of evolution of a membrane system, which is a specialization of the reaction relation introduced above. Let $\vec{R}$ and $\vec{R}'$ be two vectors multi-rules, then $\vec{R} \preceq \vec{R}'$ iff for each $1 \leq i \leq n$ $\widehat{R}_i \subseteq \widehat{R}'_i$. With $C \overset{\vec{R}}{\Longrightarrow}_{free} C'$ we indicate $C \Rightarrow C'$ using $\vec{R}$.

Definition 7 *Let* Π *be a membrane system. We define the following reaction rule on* $Conf_\Pi \times Conf_\Pi$. *Let* $(C, C') \in Conf_\Pi \times Conf_\Pi$ *and let* $\vec{R}$ *be a vector multi-rule, then* C max-evolve *into* C' $(C \Rightarrow_{max} C')$ *if* $C \overset{\vec{R}}{\Longrightarrow}_{free} C'$ *and there is no* $\vec{R}'$ *with* $\vec{R} \preceq \vec{R}'$ *and there is no* C'' *such that* $C \overset{\vec{R}'}{\Longrightarrow}_{free} C''$.

In [9] some other notions of evolution are considered. Here we focus on the notion of evolution where the maximal multiset of rules is applied, our aim being the investigation on what the events in a membrane system are and on how they are *related*. We can now formalize the notion of *reachable* configuration. With $\Rightarrow^*_{max}$ we denote the reflexive and transitive closure of $\Rightarrow_{max}$.

Definition 8 *Let* Π *be a membrane system, and* C *be a configuration. Then* C *is* reachable *iff* $C_0 \Rightarrow^*_{max} C$.

We end this section with a simple example, we will also use in the rest of the paper. Consider the following system:

$$\Pi_1 = (\{a, b, c, d\}, [_1\ [_2\ ]_2\ ]_1, c, aad, R_1 = \{r_1 = bc \rightarrow^{\emptyset}_{\emptyset} (d, here)\},$$
$$R_2 = \{r_2 = a \rightarrow^{\{d\}}_{\{c\}} (b, out), r_3 = d \rightarrow^{\emptyset}_{\emptyset} (c, here)\})$$

At the initial partial configuration $((c, \mathbf{0}), (aad, \mathbf{0}))$ the multiset with two occurrences of the rule r_2 and one of the rule r_3, can be applied and using the partial reaction relation of definition 4, we have that the system can perform two reactions obtained by the application of the rule r_2 followed by a reaction obtained by the application of rule r_3 in the internal membrane (the external membrane cannot evolve) in any order. The two rules are applied in the same maximal parallelism step (and the relation $\Rightarrow_{max}$ of definition 6 takes this into account). After the evolution under this step, two instances of object b are created by the application of rule r_2 in the outer membrane and a d in the inner membrane. Now, a further reduction step can be performed, consisting in applying rule r_1. The application of this rule depends on one of the two applications of the rule r_2 in the previous step.

4 Zero-Safe Petri Nets and 1-Unfoldings

A net is a tuple $N = (S, T, F, m_0)$ where S are *places*, T are transitions, $F : (S \times T) \cup (T \times S) \rightarrow \mathbb{N}$ is a flow relation and $m_0 : S \rightarrow \mathbb{N}$ is the initial marking. The evolution of a net is described as usual with the token game. Let $m : S \rightarrow \mathbb{N}$ be a marking of a net, a finite multiset $U : T \rightarrow \mathbb{N}$ of transitions is *enabled* under m if for all $s \in S$ $\sum_{t \in T} U(t) \cdot F(s, t) \leq m(s)$ and the reached marking is $m'(s) = m(s) + \sum_{t \in T} U(t) \cdot (F(t, s) - F(s, t))$, for all $s \in S$. We then write $m\,[U\rangle\, m'$, and call

U a *step.* A *step firing sequence* is defined as follows: m_0 is a step firing sequence, and if $m_0 \, [U_1\rangle \, m_1 \, [U_2\rangle \, m_2 \ldots m_{n-1} \, [U_n\rangle \, m_n$ is a step firing sequence and $m_n \, [U_{n+1}\rangle \, m_{n+1}$, then $m_0 \, [U_1\rangle \, m_1 \, [U_2\rangle \, m_2 \ldots m_{n-1} \, [U_n\rangle \, m_n \, [U_{n+1}\rangle \, m_{n+1}$ is a step firing sequence. A marking m is reachable if there is a step firing sequence $m_0 \, [U_1\rangle \, m_1 \, [U_2\rangle \, m_2 \ldots m_{n-1} \, [U_n\rangle \, m_n$ and $m = m_n$. Here we assume that enabling of a finite multiset is checked just once, before firing all the transitions in it. We add inhibitor and read arcs to nets, similarly to what is done in [9].[4] A net with inhibitor and read arcs is the tuple $N = (S, T, F, I, K, m_0)$ where (S, T, F, m_0) is a net and $I : T \times S \to \mathbb{N} \cup \{\infty\}$ and $K : T \times S \to \mathbb{N}$ are respectively the inhibitor and read arcs. We model the absence of inhibitor arcs putting the weight to ∞, whereas the absence of read arcs is modeled as usual setting the weight to 0. The enabling changes accordingly adding that for all $t \in T$, if $U(t) > 0$ then $I(t, s) > m(s)$ and $K(t, s) \leq m(s)$. The notion of step firing sequence does not change, as inhibitor and read arcs are used to test *side conditions.* Given a step U and a marking m, with $m \, [U\rangle$ we indicate that U is enabled under m and that there exists a marking m' such that $m \, [U\rangle \, m'$. We say that a step U is *max-enabled* under m ($m \, [U\rangle_{\,max}$), if $m \, [U\rangle$ and there is no step U', with $U \subset U'$ such that $m \, [U'\rangle$.

To be able to represent partial configurations, relevant in understanding causality relations among rule occurrences, we consider the zero-safe nets of Bruni and Montanari [5]. In these nets the set of places is partitioned into two disjoint sets, the one of *stable* places and the one of zero-safe places. The intuition is that when a zero-safe place is marked, then the state of the system is *unstable*, meaning that other transitions have still to change the state to reach a *stable* state. Thus zero safe places can be used to coordinate and synchronize in a single transaction any number of transitions in the net. The convention we use to draw zero safe places is the usual one: they are represented with smaller circles with respect the ordinary (stable) places.

Definition 9 *A* Zero safe Petri net with inhibitor and read arcs *(ZSI net) is a tuple $N = (S, T, F, I, K, m, Z)$ where*

1. *$N_s = (S, T, F, I, K, m)$ is a Petri net with inhibitor and read arcs (the* support*),*
2. *$Z \subset S$ is a subset of places, called* zero safe *places, and $S \setminus Z$ are the* stable *places, and*

[4] We do not consider, in this paper, localities to simplify the notation, but the extension is straightforward.

3. for all $z \in Z$, $m(z) = 0$.

A marking m' is said stable *iff $m'(z) = 0$ for all $z \in Z$.*

Let $m\,[U_1\rangle\, m_1\,[U_2\rangle\, m_1 \ldots m_{n-1}\,[U_n\rangle\, m'$ be a step firing sequence of N_s, $U = \sum_{i=1}^{n} U_i$ is a stable step *from m to m' if:*

- $\forall s \in S \setminus Z$
  - $\sum_{t \in T} U(t) \cdot F(s,t) \leq m(s)$,
  - *if $U(t) > 0$ then $I(t,s) > m(s)$ and $K(t,s) \leq m(s)$, and*
- *m, m' are stable markings.*

A stable step firing sequence is a step firing sequence where each step is a stable step.

In a stable step, transitions consuming and producing tokens in zero safe places can fire any number of times provided that their *stable* enabling (i.e., the enabling conditions involving only stable places) is verified before starting the sequence.

To be able to observe causality, we focus on the so called *non sequential* semantics of a net, where the causal dependencies between transition can be better perceived with respect to the step firing sequence behaviour. In [4] the collective and individual token semantics are compared in a categorical setting. Here we adopt a different approach to the collective token philosophy based on the notion of 1-occurrence net, developed by van Glabbeek and Plotkin ([17]).

We rephrase for our purposes the notion of 1-unfolding of [17] (i.e., the unfolding for the collective token philosophy) showing how a net is unfolded and then we relate the states of the 1-unfolding to reachable marking of the net. We first recall some definitions.

Definition 10 *Let N be a ZSI net, the* state *of a net is any finite multiset X of transitions with the property that the function $m_X : S \to \mathbb{Z}$ given by $m_X(s) = m(s) + \sum_{t \in T} X(t) \cdot (F(t,s) - F(s,t))$, for all $s \in S$, is a reachable marking of the net.*

An 1-occurrence net is a net such that each state is a set. Formally:

Definition 11 *Let N be a ZSI net, N is a* 1-occurrence net *(*1-ON *for short) if every state is a set.*

Given a net, its behaviour according to the collective token philosophy can be captured by the following definition ([17]).

Definition 12 *Let $N = (S, T, F, I, K, m, Z)$ be a ZSI net. Its* 1-unfolding $\mathcal{U}(N) = (S', T', F', I', K', m', Z)$ *into a 1-occurrence net is given by*

- $S' = S \cup (T' \times \{*\})$ *and* $T' = T \times (\mathbb{N} \setminus \{0\})$,
- $F'(s, (t, n)) = \begin{cases} F(s, t) & \text{if } s \in S \\ 1 & \text{if } s = (u, *) \text{ and } u \in T' \\ 0 & \text{if } s = (u', *),\ u \in T' \text{ and } u' \neq u \end{cases}$

  $F'((t, n), s) = \begin{cases} F(t, s) & \text{if } s \in S \\ 0 & \text{otherwise} \end{cases}$
- $I'((t, n), s) = \begin{cases} I(t, s) & \text{if } s \in S \\ \infty & \text{otherwise} \end{cases}$
- $K'((t, n), s) = \begin{cases} K(t, s) & \text{if } s \in S \\ 0 & \text{otherwise} \end{cases}$
- $m'(s) = \begin{cases} m(s) & \text{if } s \in S \\ 1 & \text{if } s \in (T' \times \{*\}) \end{cases}$

We stress that in the unfolding the new places (used to fire a transition just once) are never zero places.

We end this section stating the precise relation between the notion of states of the 1-unfolding and reachable markings of the net itself. Let N be a ZSI net and its 1-unfolding $\mathcal{U}(N)$. Let $W \subseteq T'$, with W_t we indicate the set $\{(t, n) |\ (t, n) \in W\}$. We define the mapping $\imath : \mathbf{2}^{T'} \to [T \to \mathbb{N}]$ as follows: $\imath(W)$ is $X(t) = |W_t|$. It is easy to see that the following propositions holds (see [14]).

Proposition 1 *Let N be a ZSI net and N' its 1-unfolding. Let X be a state of N' and m_X the reached marking, then there exists a firing sequence $m\,[U_1\rangle\, m_1\,[U_2\rangle\, m_2 \ldots m_{n-1}\,[U_n\rangle\, m_n$ in N such that $m_n(s) = m_X(s)$ for all $s \in S$ and $\sum_{i \leq n} U_i = \imath(X)$.*

Proof: By definition of state we know that m_X is a reachable marking, hence there exists a step firing sequence $m'\,[U'_1\rangle\, m'_1 \ldots m'_{n-1}\,[U'_n\rangle\, m_X$. We prove the claim by induction on the length of this sequence.

If the length is zero, the X is the empty state (empty multiset) and in N the firing sequence m is the one we need. The other parts are trivial.

Assume it holds for n. For $n+1$ we have in N' a firing sequence $m' \, [U'_1\rangle \, m'_1 \ldots m'_{n-1} \, [U'_n\rangle \, m'_n \, [U'_{n+1}\rangle \, m_X$. $X \setminus U'_{n+1}$ is a state of N' (the firing sequence leading to $m_{X \setminus U'_{n+1}} = m_n$ is $m' \, [U'_1\rangle \, m'_1 \ldots m'_{n-1} \, [U'_n\rangle \, m'_n$), and there is a firing sequence $m \, [U_1\rangle \, m_1 \ldots m_{n-1} \, [U_n\rangle \, m_n$ of N such that $m_n(s) = m'_n(s) = m_X \setminus U'_{n+1}(s)$ for all $s \in S$, and $\sum_{i \leq n} U_i = \iota(X \setminus U'_{n+1})$. Consider now the step $m'_n \, [U'_{n+1}\rangle \, m_X$, and consider the multiset of transition U_{n+1} defined as follows: $U_{n+1}(t) = |(U'_{n+1})_t|$. It is obvious that this multiset is enabled at m_n, by simply observing how the unfolding is defined, hence $m_n \, [U_{n+1}\rangle \, m_{n+1}$ and $m_{n+1}(s) = m_X(s)$ for all for all $s \in S$. This amounts to stating that there exists a firing sequence leading to a marking with the desired characteristics and furthermore $\sum_{i \leq n+1} U_i = \iota(X)$.

□

Proposition 2 *Let N be a ZSI net and N' its 1-unfolding. Let m_n be a reachable marking of N. Then there exists a state X of N' such that $m_n(s) = m_X(s)$ for all $s \in S$.*

Proof: By induction on the length of the firing sequence. If the length is zero, then the empty state is the one we are looking for and $m(s) = m_X(s)$ for all $s \in S$ by construction. Consider now $m \, [U_1\rangle \, m_1 \ldots m_n \, [U_{n+1}\rangle \, m_{n+1}$. By induction we know that there is a state X' of N' such that $m_n(s) = m_{X'}(s)$ for all $s \in S$. Consider then the marking $m_{X'}$ and the step U' defined as follows: for all $t \in T$ such that $U_{n+1}(t) = i$ and $i > 0$ take i transitions $(t, k_1), \ldots (t, k_i)$ such that $m_{X'}((((t, k_j), *)) = 1$ (as we have infinitely many copies of a transition this is always guaranteed); it is easy to observe that U' is enabled at $m_{X'}$ and furthermore that $X' \oplus U$ is a state of N'. Clearly $m_{X' \oplus U}(s) = m_{n+1}(s)$ for all $s \in S$.

□

5 From Membrane Systems to Petri Nets

In this section we show how to associate a membrane system to a zero safe Petri net and then how evolutions in membrane systems and those in a zero safe nets are related. We follow closely what has been developed by [9], adapting it to our setting. To each rule we associate a transition (which are indexed by the *name* of the rule and by the compartment), whereas places are associated to objects. In particular to each object and each membrane we associate two places, one of them being zero safe, connected by a transition consuming tokens in the zero safe place and producing them in the other one (these transitions are denoted with $t^{\hbar}_{(a,i)}$). The zero safe places are used to represent the second component of a partial configuration. The heating of a partial configuration is performed by firing the transitions $t^{\hbar}_{q}$, which can be done in a stable step. Finally, the number of tokens in a place gives the number of objects in a membrane.

Definition 13 *Let* $\Pi = (V, \mu, w^0_1, \ldots, w^0_n, R_1, \ldots, R_n)$ *be a membrane system, then we associate to it the structure* $\mathcal{F}(\Pi) = (S, T, F, I, K, m, Z)$ *where:*

- $S = V \times (\{1, \ldots, n\} \times \{nz, z\})$, $Z = V \times (\{1, \ldots, n\} \times \{z\})$, *and* $T = \bigcup_{i=1}^{n}\{t^r_i \mid r \in R_i\} \cup \{t^{\hbar}_{(a,i)} \mid a \in V \text{ and } 1 \leq i \leq n\}$,
- *for all transitions* $t = t^r_i \in T$, *with* $r = u \rightarrow^{p}_{inh} v$, *we define*

  $$F(s,t) = \begin{cases} u(a) & \textit{if } j = i \textit{ and } s = (a,(j,nz)) \\ 0 & \textit{otherwise} \end{cases}$$

  $$F(t,s) = \begin{cases} v((a, here)) & \textit{if } j = i \textit{ and } s = (a,(j,z)) \\ v((a, out)) & \textit{if } j = father(i) \textit{ and } s = (a,(j,z)) \\ v((a, in_j)) & \textit{if } j \in children(i) \textit{ and } s = (a,(j,z)) \\ 0 & \textit{otherwise} \end{cases}$$

  $$I(s,t) = \begin{cases} inh(a) - 1 & \textit{if } i = j, \quad a \in dom(inh) \textit{ and } s = (a,(j,nz)) \\ \infty & \textit{otherwise} \end{cases}$$

  $$K(s,t) = \begin{cases} p(a) & \textit{if } i = j \textit{ and } s = (a,(j,nz)) \\ 0 & \textit{otherwise} \end{cases}$$

- *for all transitions* $t = t^{\hbar}_{(a,i)} \in T$, *we define*

  $$F(s,t) = \begin{cases} 1 & \textit{if } s = (a,(i,z)) \textit{ and } t = t^{\hbar}_{(a,i)} \\ 0 & \textit{otherwise} \end{cases}$$

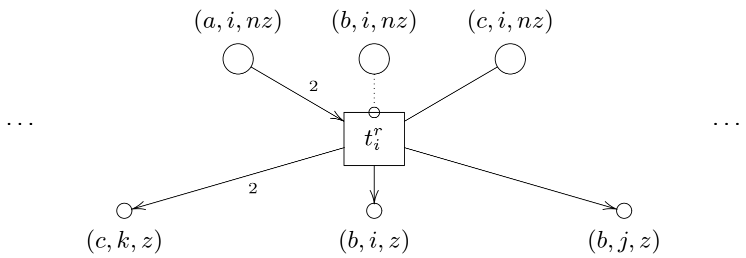
  $$F(t,s) = \begin{cases} 1 & \textit{if } s = (a,(i,z)) \textit{ and } t = t^{\hbar}_{(a,i)} \\ 0 & \textit{otherwise} \end{cases}$$

  for all $s \in S$, $I(s,t) = \infty$ *and* $K(s,t) = 0$, *and*

- $m(s) = \begin{cases} w_i(a) & \textit{if } s = (a,(i,nz)) \\ 0 & \textit{otherwise} \end{cases}$.

As we said before, the main difference with respect to other approaches is that we add a zero safe place corresponding to each object, playing the rôle of the second multiset in partial configurations (representing the objects produced while an evolution step is going on), and correspondingly we introduce a number of transition to *heat* to a stable marking.

We illustrate the construction by showing what happens in the case of the rule belonging to the set or rules associated to the membrane i, $r = aa \rightarrow_b^c (b, here)(c, out)(c, out)(a, in_j) \in R_i$. We assume that $father(i) = k$ and that $father(j) = i$. We draw only places and arcs associated to the transition associated to r which we denote with t_i^r (inhibitor arcs are drawn as dotted line with a small circle on top, read arcs as a continuous line, and if the weights is 1 then the indication of the weight is omitted).



The places in the first line ((a, i, nz), (b, i, nz) and (c, i, nz)) correspond to the objects in the membrane i (here we assume that the objects are $\{a, b, c\}$, and two tokens from the place (a, i, nz) are consumed, whereas in places (b, i, nz) and (c, i, nz) it is tested whether a token is absent, respectively present. The zero safe places in the bottom line are those that will receive tokens produced by the transition t_i^r (the indexes k, i and j, in the zero safe places $(c, k, nz), (b, i, nz)$ and (b, j, nz), denote the membrane). The tokens in the zero safe places are removed by the heating transitions.

The zero safe net corresponding to the example in section 3 is the one shown in Fig. 1.

The following proposition states that the construction in definition 13 gives indeed a ZSI net.

Proposition 3 *Let* Π *be a membrane system, then* $\mathcal{F}(\Pi)$ *is a ZSI net.*

The correspondence between partial configurations and markings is given by the following definition.

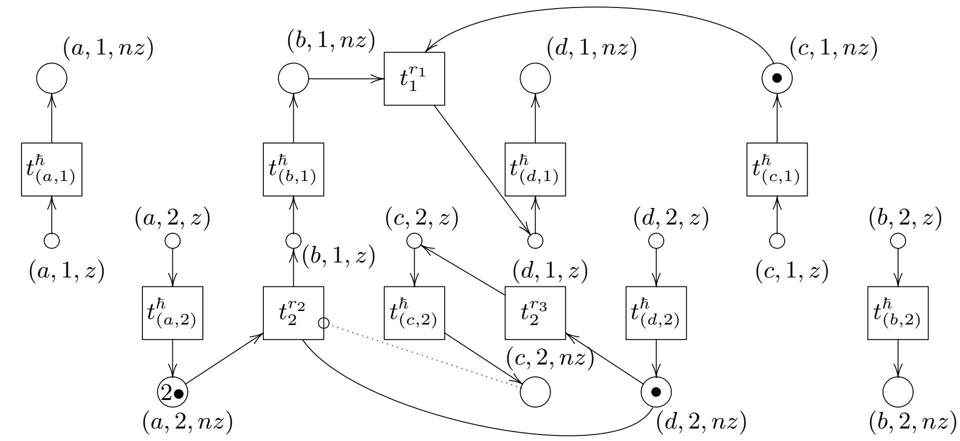


Figure 1: The ZSI net corresponding to the membrane system Π_1 of section 3.

Definition 14 *Let* Π *be a membrane system, and let* $\mathcal{F}(\Pi)$ *the associated ZSI net. Let* $C = ((w_1, \overline{w}_1), \dots, (w_n, \overline{w}_n))$ *be a partial configuration. Then the corresponding marking, denoted with* $\nu(C)$*, is given, for all* a *and* i*, by* $\nu(C)(a,(i,nz)) = w_i(a)$ *and* $\nu(C)(a,(i,z)) = \overline{w}_i(a)$*.*

With abuse of notation, given a configuration $C = (w_1, \dots, w_n)$, we write $\nu(C)$ for $\nu((w_1, \mathbf{0}), \dots, (w_n, \mathbf{0}))$.

In a membrane system, a configuration is obtained from a partial one by heating the partial configuration. In the net corresponding to a membrane system, heating means to reach a stable marking.

Definition 15 *Let* Π *be a membrane system, let* $\mathcal{F}(\Pi)$ *be the associated ZSI net and* m' *be a marking. Then* $heated_N(m')$ *is the marking where, for all* $(a,(i,nz))$*,* $heated_N(m')(a,(i,nz)) = m'(a,(i,nz)) \oplus m'((a,(i,z))$ *and, for all* $(a,(i,z))$*,* $heated_N(m')(a,(i,nz)) = 0$*.*

The following proposition relates the *heating* of a partial configuration of Π to a suitable step in the corresponding ZSI.

Proposition 4 *Let* Π *be a membrane system, and let* $\mathcal{F}(\Pi)$ *be the associated ZSI net. Let* $C = ((w_1, \overline{w}_1), \dots, (w_n, \overline{w}_n))$ *be a partial configuration and* $\nu(C)$ *the associated marking. Let* U_{heat} *be the step where, for all* $t^{\hbar}_{(a,i)}$*,* $U_{heat}(t^{\hbar}_{(a,i)}) = \overline{w}_i(a)$ *and for all other transitions* t^r_i*,* $U_{heat}(t^r_i) = 0$*.*

Then $\nu(C)\,[U_{heat}\rangle$ and $\nu(C)\,[U_{heat}\rangle\, m' = \nu(heated(C))$. Furthermore $m' = heated_{\mathcal{F}(\Pi)}(\nu(C))$.

Proof: Take the partial configuration $C = ((w_1, \overline{w}_1), \ldots, (w_n, \overline{w}_n))$. The corresponding marking $\nu(C)$ has $\overline{w}_i(a)$ tokens in the zero safe place associated to a in the membrane i, $(a, (i, z))$. The transition $t^{\hbar}_{(a,i)}$ consumes one token from the place $(a, (i, z))$ and produce one token in the place $(a, (i, nz))$. Clearly the step U_{heat} is enabled (hence $\nu(C)\,[U_{heat}\rangle$) and the effect of firing $\overline{w}_i(a)$ occurrences of the transition $t^{\hbar}_{(a,i)}$ is to add $\overline{w}_i(a)$ tokens in $(a, (i, nz))$, which will contain $w_i(a) + \overline{w}_i(a)$ tokens. Thus $\nu(C)\,[U_{heat}\rangle\, m'$ and $m' = \nu(heated(C))$. Clearly $m' = heated_{\mathcal{F}(\Pi)}(\nu(C))$.

□

Evolution steps in membrane system and in the corresponding net are related, as stated in the following propositions. We first show that the effect of a rule and the one of the application of the corresponding transition are related, and then we prove that to a step in the membrane system a stable step in the ZSI net corresponds.

Proposition 5 *Let Π be a membrane system, and let $\mathcal{F}(\Pi)$ the associated ZSI net. Let $C = ((w_1, \overline{w}_1), \ldots, (w_n, \overline{w}_n))$ be a partial configuration and $\nu(C)$ the associated marking. If $C \overset{r}{\mapsto} C'$ using $r \in R_i$ then $\nu(C)(s) + F(t^r_i, s) - F(s, t^r_i) = \nu(C')(s)$ for all $s \in S$, where t^r_i is the transition associated to r.*

Proof: Consider the rule $r = u \to^{p}_{inh} v$, to which the transition t^r_i corresponds. It is straightforward to observe that the effect of the firing of the transition t^r_i on the marking $\nu(C)$ are those in the marking $\nu(C')$. For instance, consider $w_i(a)$ and $u(a)$, as $C \overset{r}{\mapsto} C'$ we have that $w'_i(a) = w_i(a) - u(a)$ and $\overline{w}'_i(a) = \overline{w}'_i(a) + v((a, here))$, and similarly for the objects (tokens) which are sent to other membranes, as in the net this means that they are sent to the zero safe places with the proper index (i.e., the membrane). Hence the thesis.

□

Let $\vec{R} = (\widehat{R}_1, \ldots, \widehat{R}_n)$ be a vector multi-rule for the membrane system Π and let $\mathcal{F}(\Pi)$ the associated ZSI net. With $U_{\widehat{R}_i}$ we denote the multiset $U_{\widehat{R}_i}(t^{r^j_i}_i) = \widehat{R}_i(r^j_i)$. With $U_{\vec{R}}$ we denote the step $\sum_{i=1}^{n} U_{\widehat{R}_i}$.

Proposition 6 *Let Π be a membrane system, and let $\mathcal{F}(\Pi)$ the associated ZSI net. Let $C = ((w_1, \mathbf{0}), \ldots, (w_n, \mathbf{0}))$ be a partial configuration of Π and*

$\nu(C)$ *the associated marking of* $\mathcal{F}(\Pi)$. *If* $C \stackrel{\vec{R}}{\Rightarrow}_{free} C'$ *and* $\vec{R} = (\widehat{R}_1, \ldots, \widehat{R}_n)$, *then*

1. $\nu(C)\,[U_{\vec{R}}\rangle$, *and*

2. *there exist* $m_1, \ldots, m_n$ *markings such that* $\nu(C)\,[U_{\widehat{R}_1}\rangle\, m_1 \ldots m_{n-1}\,[U_{\widehat{R}_n}\rangle\, m_n$ *and* $heated_{\mathcal{F}(\Pi)}(m_n) = \nu(C')$.

Proof: (1) Let us first verify that the step $U_{\vec{R}}$ is enabled at $\nu(C)$. As $C \stackrel{\vec{R}}{\Rightarrow}_{free} C'$, $\vec{R}$ is enabled at C, then, for all i, $\sum_j \widehat{R}_i(r_i^j)(u_{ij}) \subseteq w_i$, for all i, j, $p_{i,j} \subseteq w_i$, and $w_i(a) < inh_{i,j}(a)$ for all $a \in dom(inh_{i,j})$. We first observe that the tokens to be consumed and tested (presence or absence) are tokens in non zero safe places.

Consider then $U_{\vec{R}} = \sum_{i=1}^{n} U_{\widehat{R}_i}$, clearly $U_{\vec{R}}$ is enabled at $\nu(C)$ iff

- $\sum_{i,j} U_{\widehat{R}_i}(t_i^{r_i^j}) \cdot F((a,(i,nz)),(t_i^{r_i^j})) \leq \nu(C)((a,(i,nz)),$
- $I(t_i^{r_i^j},(a,(i,nz))) > \nu(C)((a,(i,nz))),$ and
- $K(t_i^{r_i^j},(a,(i,nz))) \leq \nu(C)((a,(i,nz))).$

Now clearly $\sum_{i,j} U_{\widehat{R}_i}(t_i^{r_i^j}) \cdot F((a,(i,nz)),(t_i^{r_i^j})) \leq \nu(C)((a,(i,nz))$ corresponds to $\sum_j \widehat{R}_i(t_i^{r_i^j})(u_{ij}) \subseteq w_i$, for all i, hence it is satisfied; $I(t_i^{r_i^j},(a,(i,nz))) > \nu(C)((a,(i,nz)))$ corresponds to $w_i(a) < inh_{i,j}(a)$ as $I(t_i^{r_i^j}, s) = inh_{i,j}(a) - 1$ and finally $K(t_i^{r_i^j},(a,(i,nz))) \leq \nu(C)((a,(i,nz)))$ corresponds to $p_{i,j} \subseteq w_i$, as $K(t_i^{r_i^j},(a,(i,nz))) = p_{i,j}(a)$, hence they are satisfied. Thus $\nu(C)\,[U_{\vec{R}}\rangle$.

(2) It is easy to see that $m_i(s) = \nu(C)(s) - \sum_{l=1}^{i}\sum_j U_{\widehat{R}_l}(t_l^{r_l^j}) \cdot F((a,(l,nz)),(t_l^{r_l^j})) + \sum_{l=1}^{i}\sum_j U_{\widehat{R}_l}(t_l^{r_l^j}) \cdot F((t_l^{r_l^j}),(a,(l,nz)))$ is such that $m_i\,[U_{\widehat{R}_{i+1}}\rangle$, hence we can build the step firing sequence $\nu(C)\,[U_{\widehat{R}_1}\rangle\, m_1 \ldots m_{n-1}\,[U_{\widehat{R}_n}\rangle\, m_n$ with the m_i defined as above, and finally it is trivial to observe that $heated_{\mathcal{F}(\Pi)}(m_n) = \nu(C')$.

□

Proposition 7 *Let* Π *be a membrane system, and let* $\mathcal{F}(\Pi)$ *the associated ZSI net. Let* $C = ((w_1, \mathbf{0}), \ldots, (w_n, \mathbf{0}))$ *be a partial configuration and* $\nu(C)$

the associated marking. If $C \overset{\vec{R}}{\Rightarrow}_{free} C'$ and $\vec{R} = (\widehat{R}_1, \ldots, \widehat{R}_n)$, then there exists a stable step U such that $\nu(C)\,[U\rangle\,\nu(C')$.

Proof: By Proposition 6 we know that if $C \overset{\vec{R}}{\Rightarrow}_{free} C'$ then $\nu(C)\,[U_{\vec{R}}\rangle\, m$ and $heated_{\mathcal{F}(\Pi)}(m) = \nu(C')$. Consider then the multiset U' such that $m\,[U'\rangle\,\nu(C')$. By Proposition 4, we know that it exists and furthermore $U_{\vec{R}} \oplus U'$ is a stable step as $U_{\vec{R}} \oplus U'$ is enabled at $\nu(C)$ with respect to stable places, $\nu(C)\,[U_{\vec{R}} \oplus U'\rangle\,\nu(C')$ and $\nu(C)$, $\nu(C')$ are stable markings.

□

Thus to the evolutions in a membrane system stable step firing sequences in the associated net correspond. Hence the net mimics the possible computations of the membrane system. A possible stable step firing sequence of the net in figure 1 is $\{2t_2^{r_2}, t_2^{r_3}, 2t^{\hbar}_{(b,1)}, t^{\hbar}_{(c,2)}\}\{t_1^{r_1}, t^{\hbar}_{(d,1)}\}$ (we indicate only the multisets of transitions corresponding to the stable steps). The corresponding evolutions in the membrane system are obtained by forgetting the transitions emptying the zero safe places. Using the above propositions we have the following theorem.

Theorem 1 *Let Π be a membrane system, and let $\mathcal{F}(\Pi)$ be the associated ZSI net. Let C be a reachable configuration of Π. Then $\nu(C)$ is a reachable marking of $\mathcal{F}(\Pi)$.*

The converse holds as well, when we consider stable step firing sequences where each step is a stable step and it is maximal.

Proposition 8 *Let Π be a membrane system, and let $\mathcal{F}(\Pi)$ be the associated ZSI net. Let $C = ((w_1, \mathbf{0}), \ldots, (w_n, \mathbf{0}))$ be a configuration and $\nu(C)$ the associated marking. Let $\nu(C)\,[U\rangle\, m'$ be a stable step such that $U \neq \mathbf{0}$. Then there exists a vector multi-rule $\vec{R} = (\widehat{R}_1, \ldots, \widehat{R}_n)$ such that $C \Rightarrow_{free} C'$ using $\vec{R}$ and $\nu(C') = m'$.*

Proof: Take the stable step $\nu(C)\,[U\rangle\, m'$ of $\mathcal{F}(\Pi)$. U can be written as $U' \oplus U_{heat}$ such that

- $U'(t) > 0$ and $F(s,t) > 0$ implies that $s \in S \setminus Z$, and
- $U_{heat}(t) > 0$ and $F(s,t) > 0$ implies that $s \in Z$.

as transitions consuming tokens from stable places never consume tokens from non stable ones, and transitions consuming tokens from non stable places never consumes tokens from stable ones.

Since U is a stable step, it is clear that there exists a (possibly non stable) marking $\overline{m}$ such that $\nu(C)\,[U'\rangle\,\overline{m}$ and $\overline{m}\,[U_{heat}\rangle\,m'$ and $U = U' \oplus U_{heat}$ where U' and U_{heat} are such that if $U'(t) > 0$ then $t = t_i^{r_i^j}$ and if $U_{heat} > 0$ then $t = t^{\hbar}_{(a,i)}$. We first construct the vector multi-rule $\vec{R}$ associated to U'. $\vec{R} = (\widehat{R}_1, \ldots, \widehat{R}_n)$ is defined as follows: $\widehat{R}_i(r_i^j) = U(t_i^{r_i^j})$. Now, U' is enabled at $\nu(C)$ (as U is enabled at $\nu(C)$), hence, for all i, $\sum_j U'(t_i^{r_i^j}) \cdot F((a,(i,nz)),(t_i^{r_i^j})) \leq \nu(C)((a,(i,nz)),\ I(t_i^{r_i^j},(a,(i,nz))) > \nu(C)((a,(i,nz)))$, and $K(t_i^{r_i^j},(a,(i,nz))) \leq \nu(C)((a,(i,nz)))$. But, by definition of $\mathcal{F}(\Pi)$, the above conditions correspond to the enabling of $\vec{R}$ at C in Π. Thus $C \overset{r_1^1}{\mapsto}_{\widehat{R}_1(r_1^1)} \gamma_{\widehat{R}_1(r_1^1)} \cdots \gamma_{k-\widehat{R}_n(r_n^{l_n})} \overset{r_n^{l_n}}{\mapsto}_{\widehat{R}_n(r_n^{l_n})} \gamma_k$, as in definition 6.

By definition of U_{heat}, we have that all the transitions enabled consume tokens from zero safe place (and produce in the corresponding non zero safe ones), and it is easy to observe that $\overline{m} = \nu(\gamma_k)$, hence the thesis.

□

Let N be a ZSI net and $m\,[U\rangle\,m'$ be a step. We say that it is maximal whenever for all U' with $U \subseteq U'$ such that $m\,[U'\rangle$ (i.e., the step U' is enabled at m) it holds that $U' = U$. We can state the following theorem.

Theorem 2 *Let Π be a membrane system, and let $\mathcal{F}(\Pi)$ be the associated ZSI net. Let m be a stable marking of $\mathcal{F}(\Pi)$ reachable with a stable step firing sequence where each step is maximal. Then there exists a reachable configuration C of Π such that $\nu(C) = m$.*

Proof: Obvious, using Proposition 8.

□

Thus we have seen that to each membrane system it is possible to associate a zero safe net and to the computations of the membrane systems, stable step firing sequences of the net correspond. Furthermore, restricting the stable step firing sequence, we have seen that also the converse holds.

6 Event Automata and Membrane Systems

To introduce an event based semantics for membrane system, we recall the notion of *event automata*. An Event Automaton ([16]) basically consists of a set of states together with a transition relation. States are finite subsets of events, though they may have more structure (e.g., events may be equipped with a partial order), making the event automaton more informative; whereas the transition relation should be such that to go from a state s to a state s' an event not appearing in s should be added. Here we consider a slightly more general notion, where to go from a state to another a finite and non empty set of events should be added.

Definition 16 *An* event automaton *is a triple* $\mathbf{A} = (E, St, \rightsquigarrow)$*, where* E *is a set of* events*,* $St \subseteq \mathbf{2}^{E}_{\mathit{fin}}$ *is a set of* states *and* $\rightsquigarrow\ \subseteq St \times St$*, called the* transition relation*, satisfies: for all* $X, Y \in St$*, if* $X \rightsquigarrow Y$ *then* $Y = X \cup E'$ *with* $E' \subseteq E$*,* $E' \cap X = \emptyset$ *and* $E' \neq \emptyset$*. An event automaton* $\mathbf{A}$ *is called* simple *if* $\emptyset \in St$ *and* $\emptyset \rightsquigarrow^* s$ *for all* $s \in St$*, namely each state is reachable from the empty state, where* $\rightsquigarrow^*$ *is the transitive and reflexive closure of* $\rightsquigarrow$*.*

Clearly the empty state is considered the initial one. In [16] it is shown how the configurations of various brands of event structures (general, prime, flow) give rise to (special kinds of) event automata, thus event automata are well suited as event based model.

Event automata have been introduced to capture phenomena involving causality that are difficult to capture in other kinds of event structures, like *or-causality*, or the irrelevance of the history of an event. We illustrate this with a small example. Consider the net N in Fig. 2, each occurrence of the transition e_0 depends on either e_1 or e_2. This kind of or causality can be easily represented in event automata, and furthermore this kind of causality arises in the collective token approach to nets (we refer to [14] for a thorough discussion on the issue).

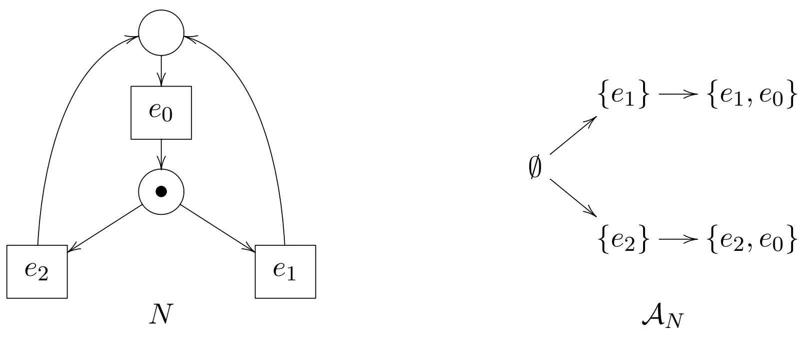


Figure 2: A net and the initial part of the associated event automaton

Given an event automaton $\mathbf{A}$, we want to consider certain states as *stable*, in complete analogy with what we have seen for ZSI net. Hence we simply *affirm* that some states are stable.

Definition 17 *An* event automaton with stable states *is the tuple* $\mathbf{A} = (E, St, \rightsquigarrow, \mathcal{S})$*, where* $\mathbf{A} = (E, St, \rightsquigarrow)$ *is an event automaton and* $\mathcal{S} \subseteq St$ *is a set of* stable *states.* $\rightsquigarrow_{\mathcal{S}} = \rightsquigarrow \cap (\mathcal{S} \times \mathcal{S})$ *are the stable transitions. We assume that* $\emptyset \in \mathcal{S}$ *always.*

We now show how to represent the computations of a membrane system as an event automaton with stable states.

Event automata are easily associated to the 1-unfolding of a net, as shown in [14]. We first introduce a suitable relation on states of the 1-unfolding of a net.

Definition 18 *Let* N *be a an* 1-ON *net, and let* $\mathcal{X}$ *be the set of states of such a net. With* $\hookrightarrow$ *we denote the following relation:* $X \hookrightarrow Y$ *iff* $X, Y \in \mathcal{X}$*,* $X \neq Y$ *and* $X \subseteq Y$*.*

Proposition 9 *Let* N *be a an* 1-ON *net, and let* $\mathcal{X}$ *be the set of states of such net. Then* $\mathcal{A}_{\mathcal{U}}(N) = (T, \mathcal{X}, \hookrightarrow)$ *is an event automaton.*

Clearly, configurations of membrane systems and states of 1-unfoldings are related (as implied by the results in the previous sections). To each reachable configuration C of the membrane system Π a reachable marking $\nu(C)$ in the net $\mathcal{F}(\Pi)$ corresponds, and to $\nu(C)$ a state of $\mathcal{U}(\mathcal{F}(\Pi))$ is associated. Therefore we have the following proposition.

Proposition 10 *Let* Π *be a membrane system and* $\mathcal{F}(\Pi)$ *the associated ZSI net. Let* $\mathcal{U}(\mathcal{F}(\Pi))$ *be the 1-unfolding of* $\mathcal{F}(\Pi)$*. Assume that* C *is a reachable configuration of* Π*. Then there exists a state* X_C *of* $\mathcal{U}(\mathcal{F}(\Pi))$ *such that* m_{X_C} *and* $\nu(C)$ *in* $\mathcal{F}(\Pi)$ *corresponds.*

Proof: We have shown that the reachable configurations are reachable markings of the corresponding ZSI net (theorem 1) and that to reachable markings of a ZSI net a state of the 1-unfolding is associated (Proposition 2), hence the thesis.

□

It is worth to notice that to a configuration more than one states in the 1-unfolding may correspond, as states encode the way such a configuration has been reached. Consider again the system following membrane system, mentioned in the introduction:

$$(\{a,b,c\}, [_1\ [_2\ ]_2\ ]_1, ab, \mathbf{0},$$
$$\{r_1 = a \to_b^{\emptyset} (a, in_2), r_2 = a \to_{\emptyset}^{c} (a, in_2)\}, \{r_3 = a \to_{\emptyset}^{\emptyset} (c, here)\})$$

To the configuration (b, c) various states can be associated, e.g., the one corresponding to the execution of r_1 and r_3 and the one corresponding to the execution of r_2 and r_3. This is in line with what was already observed by Busi in [6].

Let X be a multiset over T' and let $T \subseteq T'$, with $X|_{T'}$ we indicate the multiset over T such that $X|_T(t) = X(t)$ for all $t \in T$. Consider now $\mathcal{U}(\mathcal{F}(\Pi))$, associated to the membrane system Π. The states of the 1-unfolding comprise also the transitions used for heating a marking, which we can simply forget, as we are interested in the events associated to the transitions corresponding to occurrences of rules. The states of the 1-unfolding of a ZSI net can be characterized as *stable* in the case the associated marking is stable. This means that, in the case of the 1-unfolding $\mathcal{U}(\mathcal{F}(\Pi))$, to a state X and to its m_X associated marking a stable marking m in the net $\mathcal{F}(\Pi)$ corresponds (Proposition 1), and to this stable marking m a configuration C corresponds (each $w_i(a) = m((a,(i,nz))))$. Notice that if the reachable marking m of a net N is stable then the marking m_X associated to m (corresponding to a state X of $\mathcal{U}(N)$) is stable, and vice versa (i.e., given X a state such that m_X is stable, then the corresponding marking m is stable). Let us fix the notation for the following. Define $\mathcal{S} = \{X \mid X \in \mathcal{X}$ and m_X is a stable marking$\}$, and, for a set of multiset over T', $\mathcal{X}$, $\mathcal{X}|_T = \{X \mid \exists Y \in \mathcal{X}$ and $X = Y|_T\}$, where $T \subseteq T'$. We can now associate an event automaton to a membrane system, obtaining an event structure semantics for them.

Theorem 3 *Let Π be a membrane system, let $\mathcal{F}(\Pi))$ and $\mathcal{U}(\mathcal{F}(\Pi))$ the associated ZSI and 1-unfolding. Let $\mathcal{X}$ ($\mathcal{S}$) be the set of states (respectively stable state) of $\mathcal{U}(\mathcal{F}(\Pi))$, and let T_r be the set of transitions of $\mathcal{U}(\mathcal{F}(\Pi))$ corresponding to transition in T associated to rules of Π. Then $\mathbf{A}_\Pi = (\mathcal{X}|_{T_r}, T_r, \rightsquigarrow, \mathcal{S}|_{T_r})$ is an event automaton such that to each configuration $C \in Conf_\Pi$ a reachable stable state X_C in $\mathcal{X}$ corresponds.*

Proof: The proof follows from Proposition 10.

□

The automaton corresponding to the membrane system Π_1 (and obtained by the 1-unfolding of the associated net) has the following stable states (here we identify all the possible instances of a transition with the only occurrence of this transition): $\emptyset$ (corresponding to the $\mathbf{0}$ multiset),

$\{(t_1^{r_2},1),(t_2^{r_2},2),(t_2^{r_3},1)\}$ and $\{(t_2^{r_2},1),(t_2^{r_2},2),(t_2^{r_3},1),(t_1^{r_1},1)\}$ (other states, unstable, are e.g., $\{(t_2^{r_2},1)\}$ and $\{(t_2^{r_3},1)\}$). As it will be discussed briefly in the next section, the dependency between $(t_1^{r_1},1)$ and $(t_2^{r_2},1)$ or $(t_2^{r_2},2)$ is not immediately modeled in event automata using a relation over *events* (in fact, $(t_1^{r_1},1)$ uses one b produced by $(t_2^{r_2},1)$ or $(t_2^{r_2},2)$). The automaton is the one in Fig. 3, where non stable states are underlined.

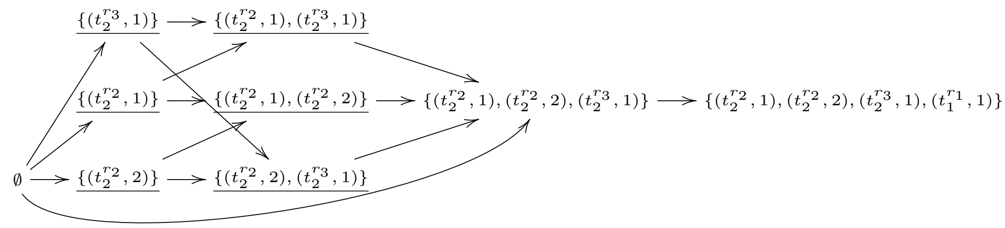


Figure 3: The event automaton associated to the 1-unfolding of the net in Fig. 1

Let us consider again one of the examples of the introduction, namely the membrane system $\Pi_2 = (\{a,b,c\}, [_1\ [_2\ ]_2\ ]_1, ab, \mathbf{0}, \{r_1 = a \to_b^\emptyset (a, in_2), r_2 = a \to_\emptyset^c (a, in_2)\}, \{r_3 = a \to_\emptyset^\emptyset (c, here)\})$. To this membrane system the ZSI net in Fig. 4 is associated.
Its 1-unfolding is the one depicted in Fig. 5 (we draw only one copy of each transition of the ZSI net in Fig. 4).

To this 1-unfolding the following event automaton, which pinpoints the or causal dependence of r_3 from r_2 and r_1, is associated. All the states of the automaton are stable:

$$\begin{tikzcd} & \{t_1^{r_2}\} \arrow[r] & \{t_1^{r_2}, t_2^{r_3}\} \\ \emptyset \arrow[ur] \arrow[dr] & & \\ & \{t_1^{r_1}\} \arrow[r] & \{t_1^{r_1}, t_2^{r_3}\} \end{tikzcd}$$

We end this section observing that two important properties of the causal semantics introduced by Busi in [6], namely

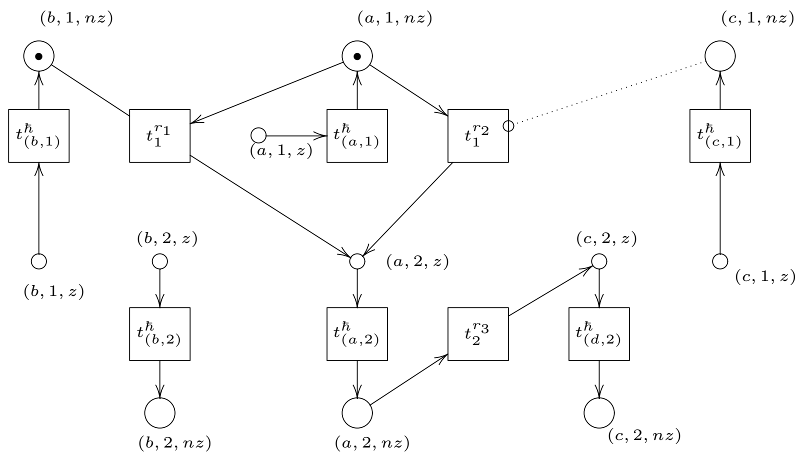


Figure 4: The ZSI net associated to Π_2

- retrievability of the maximal parallelism step semantics (from event automata it is possible to retrieve the computation in the membrane system obeying to the maximal parallel rule), and

- diamond properties, i.e., two non causally related events can happen in any order

are valid in our setting as well. For the first one it is enough to concentrate the attention on the computations involving only stable states, whereas for the second the construction of the 1-unfolding suffices.

7 Discussion

In this paper we have presented two results. The first one associates to each membrane system a zero safe net. Following the notion of partial configuration introduced by Nadia Busi, we argued that an evolution step in a P system can be represented as a suitable transaction in a ZSI net, where the zero places are used to *synchronize* the various rules applied in a step. The second one presents an *unfolding* of the ZSI net associated to the membrane system as an event automaton, which is a rather abstract way to show the

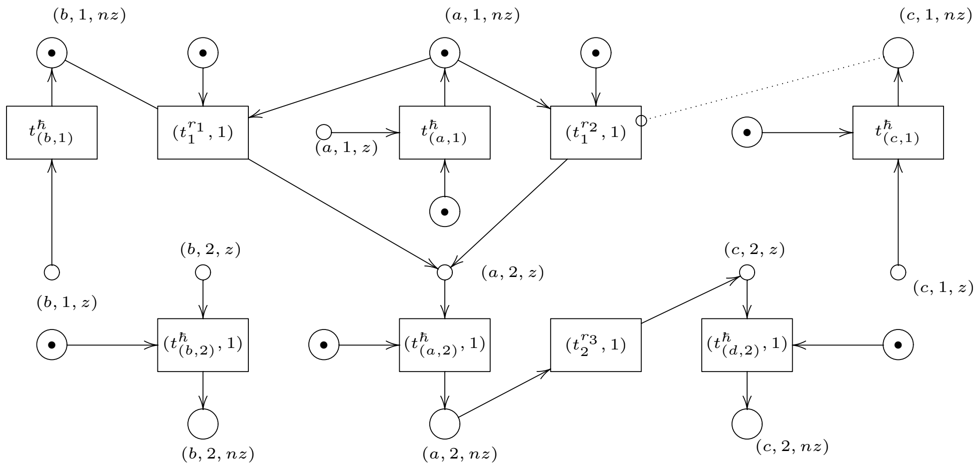


Figure 5: The 1-unfolding of the ZSI net in Fig. 4

configurations of an event structures and how it is possible to reach a configuration from another. With respect to other approaches relating nets and membrane systems, the one we propose has the advantage of considering the so called collective tokens philosophy. As we said before, this philosophy represents more faithfully, in our opinion, the multiset approach in membrane system: objects produced by different instances of the same rule in the same step are indistinguishable from a causality point of view. On the other hand, the main drawback of event automata is that dependencies among events are not represented explicitly. Here we briefly discuss some future research direction, which can lead to what we would like to obtain, i.e., a direct way to associate a suitable event structure to the 1-unfolding of a net.

In [1, 2] the *disabling/enabling* relation has been introduced to take into account the causalities arising in Petri nets with read and inhibitor arcs. This relation can be used to model dependencies between activities in a much broader sense, as shown in [15], where also a further extension of the definition has been proposed. Here we briefly recall this definition. A *disabling/enabling* relation over a set E of events, *(DE-relation* for short*)* is a ternary relation $\Im \subseteq \mathbf{2}^E_{fin} \times E \times \mathbf{2}^{\mathbf{2}^E_{fin}}$. Informally, if $\Im(a, e, A)$ then the event in a inhibits the event e, which can be enabled again by one of the set of events in A. The first argument of the relation can be also the empty set $\emptyset$, $\Im(\emptyset, e, A)$ meaning that the event e is inhibited in the initial state of the system, thus some other event should happen before e. Moreover the third argument (the set of set of events A) can be empty, $\Im(a, e, \emptyset)$ meaning that there are no events that can re-enable e after it has been disabled by a. With these ingredients, the notion of DE-event structure can be introduced: a *DE-event structure* (DE-ES) is a pair $\mathbf{E} = \langle E, \Im \rangle$, where E is a set of *events* and $\Im \subseteq \mathbf{2}^E_{fin} \times E \times \mathbf{2}^{\mathbf{2}^E_{fin}}$ is a ternary relation called *disabling-enabling relation (DE-relation* for short*)*, and clearly a computation is presented as a *configuration*, i.e., a set of events. The interesting result is that to each event automaton a DE-ES can be associated, as shown in [15]. Thus we can describe in a more fruitful way the dependencies among the events.

It should be stressed that here events are application of rules, but another reasonable approach would be to consider the changing of a configuration as provoked by the happening of an (possibly compound) event. Thus the $\Im$ could be changed by allowing that the second component can be a non empty set of events, and in this way a more precise account of the evolution of membrane system can be done. Furthermore we believe that this relation can be used to obtain a DE-ES directly from an 1-unfolding of a zero safe net.

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